ABSTRACT

We describe a modification to existing algorithms for the calculation of FORC diagrams using locally-weighted regression smoothing (often referred to as ‘loess’ smoothing). Loess smoothing offers several advantages over current methods:

1. It allows the FORC distribution to be calculated using a constant smoothing factor all the way to the \( H = 0 \) axis, without the need for a ‘reversible ridge’ which can swamp the low-coercivity signal and introduce unwanted artefacts.

2. It allows much finer control over the degree of smoothing, enabling a graphical method for automated selection of the optimum smoothing factor for a given FORC measurement.

3. It performs automated extrapolation across gaps or undefined regions of FORC space. This has two applications: a) bad curves or outlying data points caused by instrumental instabilities can be removed from the data, eliminating artefacts from the final FORC diagram; b) specific regions of interest in the FORC measurement can be masked out in order to investigate their contribution to the final FORC distribution.

The new method has been implemented in a freely available software package "FORCinel" which can be downloaded from http://www.esc.cam.ac.uk/~rjh40/forcinel. FORCinel is a suite of functions with a graphical user interface written using Igor Pro by WaveMetrics. A fully functional demo version of Igor Pro for Macintosh and Windows can be downloaded from www.wavemetrics.com.

METHOD

Like conventional smoothing algorithms, loess smoothing fits a second degree polynomial to the measured magnetisation surface over a specified area in FORC space defined by a smoothing factor. This method differs from conventional algorithms in two ways: firstly the area of FORC space used for fitting is defined as a region of arbitrary shape encompassing N nearest-neighbour data points; secondly, data inside the fit region are given a weight that depends on the distance of each data point from the point where the FORC function is being evaluated: data closer to the point being evaluated have higher weights and have a greater effect on the fit.

In the conventional FORC smoothing algorithm a smoothing factor SF corresponds to a box containing N = (2SF + 1)² points. All points are given equal weight in the fit (Fig. 1a). In loess smoothing a region of FORC space containing N points is defined. This region may be centred on and/or span across gaps or undefined regions of FORC space. Points are weighted using a ‘tri cubic’ function:

\[
\psi(x) = \left( 1 - \frac{x - x_i}{\max(x - x_i)} \right)^3
\]

where \( x \) defines the position where the FORC function is to be evaluated and \( x_i \) defines the position of the data point.

FORC DISTRIBUTION AT \( H_c = 0 \)

The loess smoothing algorithm allows the FORC distribution to be calculated at constant smoothing factor all the way to the \( H = 0 \) axis without the need for the ‘extended FORC’ or ‘reversible ridge’ formalism (Pike 2003), which can obscure the low-coercivity region of the FORC diagram and lead to unwanted artefacts. A typical example is shown in Fig. 2. The sample is an ocean basalt sample from Ocean Drilling Program Leg 206 Site 1256 measured by Chris Scudder (downloaded from FORCopedia http://forc.ucdavis.edu/forcopedia.html). On the left is the FORC diagram processed using the reversible ridge formalism (using FORCit software, http://paleomag.geology.ucdavis.edu/software-forcit.html). Ideally the reversible ridge is a delta function on the \( H = 0 \) axis. However, due to smoothing the ridge often penetrates into the low-coercivity region, creating artefacts. On the right the loess smoothed version shows ideal SD behaviour with no edge effects.

DATA MASKING

This panel illustrates the ability to extrapolate across gaps in the data. The data are from a micromagnetic simulation of PSD reversal in a 150 nm magnetite cube. Three FORCs show anomalous behaviour relating to a switch from vortex to single domain states (*). Remove these three curves from the data leads to the suppression of negative (blue) peaks in the final FORC diagram.

Fig. 1a
Conventional smoothing with \( N = 9 \)

Fig. 1b
Loess smoothing with \( N = 9 \)

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Fig. 2a
Reversible Ridge

Fig. 2b
Loess

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Fig. 3
Residual plots of observed minus smoothed magnetisation

Fig. 4a
shows a plot of the standard deviation of the fit residual, \( \sigma_r \), as a function of the smoothing factor. In the limit of no smoothing \( \sigma_r = 0 \), as the observed and fitted magnetisation surfaces are identical, \( \sigma_r \) increases rapidly with increasing smoothing as the noise in the experimental data is eliminated. The optimal degree of smoothing is reached when noise is removed without biasing the fit to the underlying surface. This corresponds to a minimum in \( d\sigma_r/dSF \) (Fig. 4b). Beyond the optimum smoothing factor, \( d\sigma_r/dSF \) rises once more due to increasing bias in the fit to the magnetisation surface. This bias is revealed by the presence of distinct features in the residual plot (Fig. 3b).

A reasonable empirical estimate of the optimum smoothing factor is obtained by calculating the position of the minimum in the \( d\sigma_r/dSF \) curve (Fig. 4b). In the example here (a pyroxene crystal containing magnetic needles), the minimum is rather flat, indicating that values of SF between 3 and 3.5 are acceptable.

Fig. 4a
(a) Standard deviation of the fit residual (\( \sigma_r \)) versus smoothing factor SF. (b) \( d\sigma_r/dSF \).